or

$$\frac{E_1}{E_x} = \cos^4\theta + a\sin^4\theta + b\sin^2 2\theta \quad (8)$$

with

$$a = E_1/E_2, \quad \text{and} \quad b = \frac{1}{4}(E_1/G_{12} - 2\nu_{12}) \quad (9)$$

Equation (8) may be written as

$$\frac{E_1}{E_x} = (1 + a - 4b)\cos^4\theta + (4b - 2a)\cos^2\theta + a \quad (10)$$

Differentiating E_1/E_x with respect to θ, and setting $d(E_1/E_x)/d\theta$ to zero, we have

$$\frac{d(E_1/E_x)}{d\theta} = -4(1 + a - b)\cos\theta\sin\theta - 2(4b - 2a)\sin\theta\cos\theta = 0 \quad (11)$$

$$= \sin\theta\cos\theta[4(4b - a - 1)\cos^2\theta - 2(4b - 2a)] = 0 \quad (12)$$

Equation (12) indicates that there are relative maximum and minimum values of E_1/E_x at $\theta = \pi/2$ and $\theta = 0$, which are obvious. In addition there may exist an absolute maximum or minimum value of E_1/E_x at

$$\theta = \cos^{-1}[(2b - a)/(4b - a - 1)]^{1/2} \quad (13)$$

For $0 < \theta < \pi/2$, either of the following conditions must be met

$$4b - a - 1 > 2b - a > 0 \quad (14)$$

or

$$4b - a - 1 < 2b - a < 0 \quad (15)$$

If we substitute Eq. (9) back into the inequalities (14) and (15), we have the following criteria:

$$G_{12} < \frac{E_1}{2(E_1/E_2 + \nu_{12})} \quad \text{or} \quad G_{12} > \frac{E_1}{2(1 + \nu_{12})} \quad (16)$$

It is interesting to note that inequalities (16) bear some imilarity to the well-known $G = E/[2(1 + \nu)]$ for isotropic materials. It can be further proved that if $G_{12} < E_1/[2(E_1/E_2 + \nu_{12})]$, an absolute minimum value E_x which is less than E_2 exists, on the other hand if $G_{12} > E_1/[2(1 + \nu_{12})]$, an absolute maximum value of E_x which is greater than E_1 exists. If G_{12} is within the range of $E_1/[2(E_1/E_2 + \nu_{12})]$ and $E_1/[2(1 + \nu_{12})]$, then E_1 and E_2 represent the absolute maximum and minimum values of Young's modulus of the orthotropic material.

The composite material described in the beginning of this note has a shear modulus, $G_{12} > E_1/[2(1 + \nu_{12})]$, therefore, an absolute maximum value of E_x exists. Further examples were also found in the literature. In Ref. 2 the example shows the material of which the shear modulus falls between $E_1/[2(1 + \nu_{12})]$ and $E_1/[2(E_1/E_2 + \nu_{12})]$; Fig. 1.5 (p. 10) indicates that E_1 and E_2 are the absolute maximum and minimum values of E_x. In Ref. 3, the figure (p. 57) shows the variation of E_x vs θ of a boron-epoxy lamina, and indicates the existence of an absolute minimum value less than E_2. Even though the elastic moduli of the material were not given in that reference, it is conceivable that for a single lamina of boron composite the shear modulus G_{12} is very low, and consequently $G_{12} < E_1/[2(E_1/E_2 + \nu_{12})]$ is satisfied.

In Ref. 4 in which anisotropic strength of composites was investigated, it was reported that a minimum normal strength exists between $\theta = 0$ and $\pi/2$ when $T^2 \ll X^2$ and $Y > (2T)^{1/2}$. This phenomenon is similar to the variation of Young's modulus with respect to θ as reported in the present note. It is also noteworthy that there may exist an absolute maximum normal strength between $\theta = 0$ and $\pi/2$ when $T > X$, and this happens when the composite material is laminated in such a way that the shear strength is greater than the normal strength in the major principal direction.

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A New Oxidizer for Auxiliary Ignition in Liquid Rocket Motors

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Introduction

KINETIC and mechanistic studies of combustion reactions are sometimes helpful in locating important steps which govern the combustion processes. A correct knowledge of the combustion mechanism can suggest various possible modifications which could be made in order to improve the performance of a particular system. Very few such studies have been carried out[1–5]. Rastogi and co-workers have intensively studied the kinetics and mechanism of the aniline and RFNA system[1–3, 5]. The mechanistic study shows that both oxidation and nitration of aniline take place during combustion. Thus the combustion of aniline can be facilitated either by accelerating the oxidation route or the nitration route or both. The present studies were undertaken to discover new systems which could enhance the ignition process. The (RFNA + SO_2)/aniline system was investigated since the oxidizer system promotes quicker nitration[6].

Experimental

Preparation of oxidizer

The oxidizer system was prepared by bubbling SO_2 gas into RFNA kept in an ice bath for approximately 5–6 hours. The density of (RFNA + SO_2) mixture and RFNA was 1.6877 and 1.4032 respectively at 20°C. Aniline was purified by twice distilling it over zinc dust with the aid of a fractionating column.

Measurement of ignition delay

Ignition delay for the new system was measured by the oscilloscope method described earlier.[7] The results are given in Table 1. The uncertainty in the measurement was of the order of ±.05 sec. However, the ignition delay for RFNA-aniline system was measured by cup-test method.

Table 1 gives the ignition delay of combustion of aniline with (RFNA + SO_2) for various values of oxidizer-fuel ratio.

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Table 1 Ignition delay data of RFNA-aniline and ($RFNA + SO_2$)-aniline systems

	Temperature = 20 ± 2°C Ignition delay, sec	
Mixture ratio, O/F	RFNA-aniline	($RFNA + SO_2$)-aniline
0.50	No ignition	1.00
0.84	No ignition	0.60
1.00	No ignition	0.20
1.34	20.9	0.15
1.67	8.0	0.05
2.67	4.8	0.05
3.34	6.6	0.10

It is obvious that ignition delay of this system is considerably reduced with respect to RFNA aniline.

($RFNA + SO_2$) may not be good as an oxidizer since it contains sulphur which is likely to increase the average molecular weight of the combustion products and consequently it may reduce the efficiency of the propellant system, but it would certainly be very useful as oxidizer for auxiliary ignition.[8]

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Nonorthogonal Coordinates

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1.0 Introduction

A VECTOR as defined in classical mechanics is a special quantity which has a magnitude and a direction

$$\bar{x} = x_{1A}\bar{\imath}_A + x_{2A}\bar{\jmath}_A + x_{3A}\bar{k}_A$$
$$= x_{1B}\bar{\imath}_B + x_{2B}\bar{\jmath}_B + x_{3B}\bar{k}_B \qquad (1.1)$$

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where $\bar{\imath}_A, \bar{\jmath}_A, \bar{k}_A$ and $\bar{\imath}_B, \bar{\jmath}_B, \bar{k}_B$ are base vectors associated with coordinate frames A and B respectively. x_{1A}, x_{2A}, x_{3A} and x_{1B}, x_{2B}, x_{3B} are coefficients of the vector x associated with each of the base vectors.

A dyadic is a special quantity having a magnitude and two associated directions defined as follows: $\bar{\bar{I}} = I_{11}\bar{\imath}\bar{\imath} + I_{12}\bar{\imath}\bar{\jmath} + I_{13}\bar{\imath}\bar{k} + I_{21}\bar{\jmath}\bar{\imath} + I_{22}\bar{\jmath}\bar{\jmath} + I_{23}\bar{\jmath}\bar{k} + I_{31}\bar{k}\bar{\imath} + I_{32}\bar{k}\bar{\jmath} + I_{33}\bar{k}\bar{k}$.

A matrix is an array of elements arranged in some systematic manner. The theory of matrices are simply rules established for the mathematical operation of these large arrays. In classical mechanics, one often has to deal with large sets of vector equations. It will be advantageous to apply matrix theory to facilitate rapid and systematic manipulation of these equations. Thus far, most of the application of matrix techniques involved the transformation of all vector and dyadics into one basic coordinate frame so that their numerical values commune. In what follows, slight modifications and additions are made to the basic matrix theory to ease its application in handling vector equations in classical mechanics. Specifically, we shall introduce matrices whose elements are basis vectors. To prevent confusion, a column or row array of elements will be referred to in this paper as column and row matrices. The word vector is reserved exclusively for the notation of a quantity having a magnitude and a direction.

2.0 Basic Definitions and Notation

2.1 Notations

All lower case letters with a bar are used to denote Euclidean vectors, defined on a three dimensional Euclidean Vector Space. Lower case letters with no bar denote scalars. Parentheses are used to denote a 3 × 1 matrix array.

This symbol $(\bar{e}_A)$ is used to denote a set of three base vectors of coordinate system A, namely,

$$(\bar{e}_A)^T = (\bar{e}_{1A}, \bar{e}_{2A}, \bar{e}_{3A}) \qquad (2.1)$$

Brackets are used to denote a square matrix. The tilde matrix $[\tilde{c}]$ is the 3 × 3 skew-symmetric matrix associated with a particular 3 × 1 row matrix $(c)^T = (c_1, c_2, c_3)$ defined as follows.

$$[\tilde{c}] = \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix} \qquad (2.2)$$

2.2 Special definitions

2.2.1 Matrix representation of a vector. Every vector $\bar{x}$ of a three dimensional vector space can be expressed in terms of a Base Vector Matrix $(\bar{e}_A)$, and a Component Matrix, (x_A), in the form given by Eq. 2.3 as follows:

$$\bar{x} \triangleq (\bar{e}_A)^T(x_A) = (x_A)^T(\bar{e}_A)$$
$$= x_{1A}\bar{e}_{1A} + x_{2A}\bar{e}_{2A} + x_{3A}\bar{e}_{3A} \qquad (2.3)$$

It is seen that Eq. 2.3 is consistent with the Cartesian representation of a vector.

2.2.2 Matrix representation of a dyadic

$$\bar{\bar{I}} = (\bar{e}_A)^T[I_A](\bar{e}_A) \qquad (2.4)$$

Every dyadic can be represented in terms of a Component Matrix, $[I_A]$, and a set of base vectors in a form given by Eq. 2.4. The component matrix $[I_A] = [I_{ijA}]$ is the 3 × 3 matrix formed from the nine components of $\bar{\bar{I}}$ with respect to $\bar{e}_{iA}\bar{e}_{jA}$.

2.3 Fundamental Operations

Two fundamental mathematical operations with the base vector matrices are defined below. These two fundamental operations are the basis for formalizing all algebraic operations with vectors and dyadics.